CS222 / Phil358 Homework 2

Due: May 5, 2010

1. This question asks you to prove one direction of Theorem 14.2.2. of the *Multiagent Systems* text. Suppose $\mathcal{M} = (W, \leq, V)$ is a KB-model, and define the belief set $K := \{\varphi : \mathcal{M} \vDash B\varphi\}$. Finally we define a revision operator $*$ so that $K * \psi := \{\varphi : \mathcal{M} \vDash B^{\psi}\varphi\}$. Show that $*$ satisfies the AGM postulates for the belief set K.

2. Recall the "mass upgrade" iterated belief revision operator takes a KB-model $\mathcal{M} = (W, \leq, V)$ and a formula φ to a new model $\mathcal{M}^{\varphi} = (W, \leq^{\varphi}, V)$, in which the ordering among φ-worlds remains unchanged, as does the ordering among $\neg\varphi$-worlds, but all φ worlds become preferred to all $\neg\varphi$-worlds. That is, $\leq^{\varphi}$ is defined so that:

(i) If $x \leq y$ and $\mathcal{M}, x \vDash \varphi$, then $x \leq^{\varphi} y$;

(ii) If $x \leq y$ and $\mathcal{M}, y \nvDash \varphi$, then $x \leq^{\varphi} y$;

(iii) If $x \leq y$, $\mathcal{M}, x \vDash \varphi$, and $\mathcal{M}, y \vDash \varphi$, then $x \leq^{\varphi} y$.

Verify it is possible to model Darwiche and Pearl's "red bird" example, without retracting the observation that the object is red. That is, the agent should first assume the object is neither red nor a bird, then learn it is a bird, then that it is red, then that it is not a bird, and in the end conclude it is red but not a bird. You should only need a model with four worlds.

Is it possible to set up the initial model so that the "red" observation is instead retracted, still using mass upgrade? Why or why not?

3. The "elite upgrade" operator takes a KB-model $\mathcal{M} = (W, \leq, V)$ and formula φ, and gives a new model $\mathcal{M}^{\varphi} = (W, \leq^{\varphi}, V)$, in which $\leq^{\varphi}$ is just like $\leq$, except that the $\leq$-minimal worlds validating φ are the absolute $\leq^{\varphi}$-minimal worlds. Prove that property # holds for any $\mathcal{M}$.

$$(\#) \text{ If } \varphi \vdash \neg\psi, \text{ then } (\mathcal{M}^{\psi})^{\varphi} \vDash B\chi, \text{ iff } \mathcal{M}^{\varphi} \vDash B\chi, \text{ for all } \chi.$$

This shows that for elite upgrade, we have $(K*\psi)*\varphi = K*\varphi$, whenever $\varphi \vdash \neg\psi$.

4. Suppose Ψ is a pedigreed belief state over W and $\sqsubset$ is a strict linear order on a set $\mathcal{S}$ of belief states over W (i.e. a set of pairs $(W, \leq)$ for some total preorder $\leq$). Prove that the ordering induced by $\Psi_{\sqsubset}$ is a total preorder on W.